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Radar Waveforms for Suppression of Extended Clutter

NOVEMBER 1966

Prepared by A. W. RIHACZEK and R. L. MITCHELL
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AEROSPACE CORPORATION

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Prepared for BALLISTIC SYSTEMS AND SPACE SYSTEMS DIVISIONS
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
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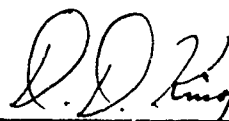
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FOREWORD

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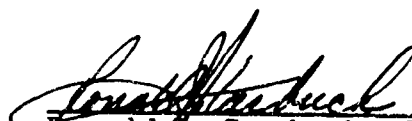
This report, which documents research carried out from 1 February to 15 June 1966, was submitted on 22 November 1966 to Captain Ronald J. Starbuck, SSTRT, for review and approval.

Approved



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Electronics Research Laboratory
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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.



Ronald J. Starbuck, Captain
Space Systems Division
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ABSTRACT

Target detection in the clutter from extended fields of scatterers, such as the ground or the sea, is a common problem in radar. Having optimized the system configuration, the designer can further improve radar performance by adapting the transmitted waveform to the particular target environment. This problem is investigated here for detection of a single target in the presence of an extended clutter space. The paper considers the possibility of confining the matched-filter response in delay and Doppler, or ambiguity function, to a narrow strip with arbitrary orientation in the delay-Doppler domain. It is shown that strict confinement of the response is achievable only with waveforms that are unlimited in both time and frequency domain. In practice, efficient use of the frequency band requires that the spectrum be truncated, so that strict confinement of the response is not achievable. One finds that bandwidth can be traded against visibility in clutter. More generally, for a fixed bandwidth, the trade-off is between close-target separability and detectability in the clutter, which are the two tasks that together constitute the resolution problem. The paper illustrates the effects of spectrum truncation for the important case of maximum confinement of the ambiguity function.

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I. INTRODUCTION

Radar clutter generally appears in two forms. In the situation implied by the more conventional meaning of the term, clutter is the return from scatterers that are of no interest to the radar user. In the second case, the term clutter is used to describe the mutual interference between targets that may all be equally important. This type of clutter is due to the fact that in a high-resolution system the response to any one target extends over many resolution cells. It is the familiar problem of sidelobes in a pulse compression radar. Both types of clutter are equally detrimental in their effect on radar performance.

Signal detection in clutter has been studied widely in the past. Early work deals with radar having a resolution capability only in range and considers filter design for clutter suppression [1,2]. With the advent of radar using sophisticated signals, waveform design as a means for clutter suppression became of interest [3-5]. This approach is more powerful, and the problem is one of adapting the waveform to the target environment. Filter optimization is useful only when the radar uses, for some reason, a waveform that is actually unsuited for the task [6]. These general discussions on waveform design and detection performance in clutter are supplemented in this paper by a specific approach to the suppression of interference from extended clutter spaces. We consider waveforms whose matched-filter responses in delay and Doppler are essentially confined to

narrow strips in the delay-Doppler plane. With proper orientation of the strip, a waveform of this type will enhance target detectability in clutter.

The scope of the study is that of the conventional theory of resolution in range and range rate, based on Woodward's ambiguity function [7]. Hence, it is implicit that range acceleration (and higher-order range derivatives) has negligible effects over the duration of the signal, and that distortions of the modulation function due to range rate can be neglected. Practically, this amounts to limiting the study to range rates $\dot{R} \leq 0.1c/BT$ and range accelerations $\ddot{R} \leq 0.1\lambda/T^2$, where c is velocity of light, B is signal bandwidth, T is duration, and λ is wavelength.

II. WAVEFORM OPTIMIZATION

Consider the problem of recognizing a particular target return in the interference from a large number of scatterers distributed in range and range rate, or in range delay and Doppler. Let us assume that those scatterers actually illuminated by the same radar pulse (and hence contributing to the clutter) are contained within some area in the delay-Doppler plane, as indicated in Fig. 1. We shall call this area the clutter space and, in specializing the study, assume that the area has a size large compared with unity. This is what we mean by an "extended" clutter space. The target of interest may be within the clutter space or outside its boundaries. The question is how the waveform should be designed to minimize the interference from the clutter space.

The requirements on the waveform for elimination of the interference from the clutter space are well understood [4, 5]. The associated ambiguity function must have a shape such that when its central peak is positioned anywhere within the clutter space, its magnitude at the target position A is zero. This goal is principally unattainable when the clutter space is extended, if the target is within the clutter space. The best that can be done is to minimize the interference by proper design of the waveform. In theory, complete suppression of the clutter is possible when the target is outside the clutter space, but then practical considerations limit the achievable performance.

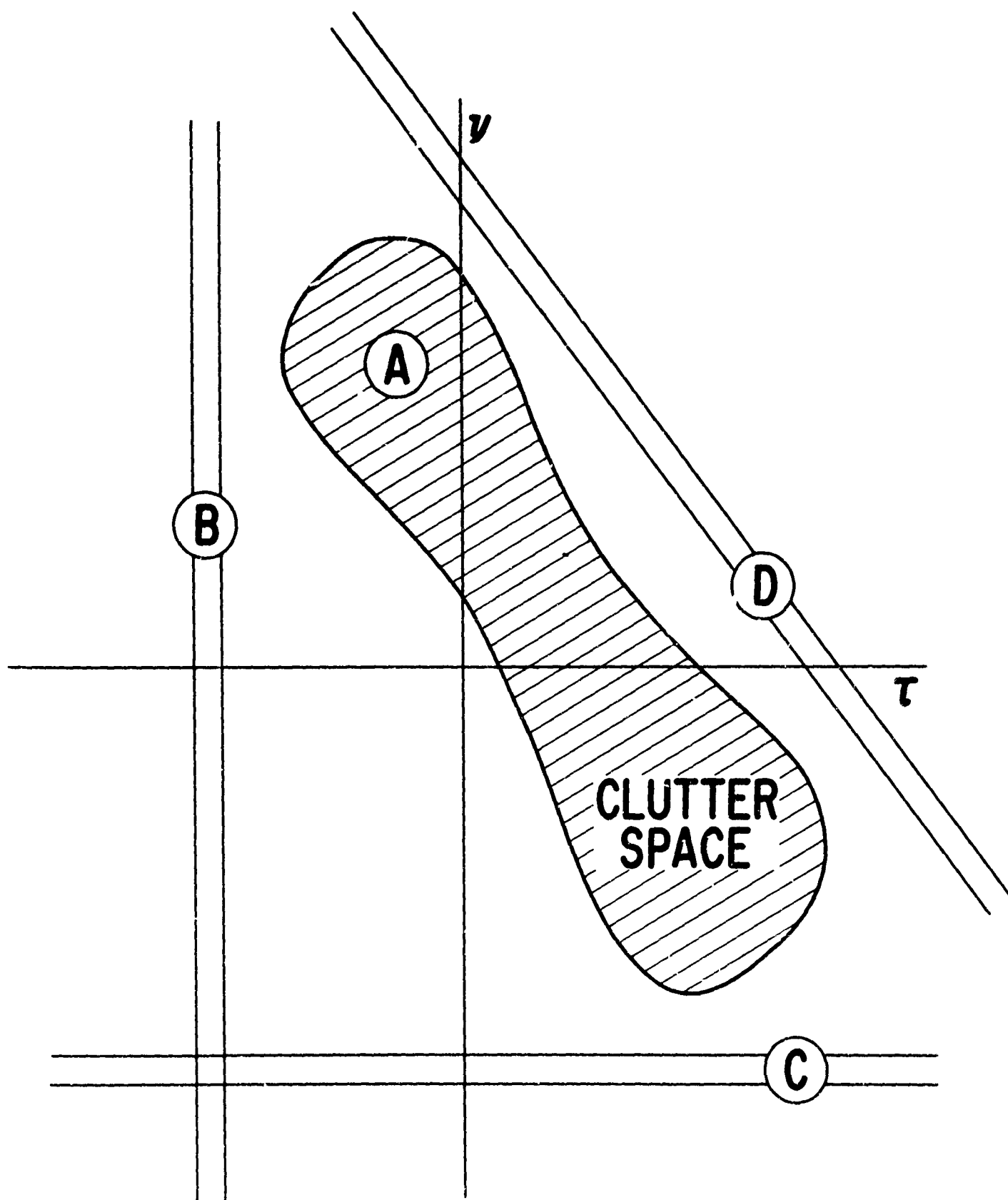


Fig. 1. Clutter space and ambiguity functions with clutter suppression properties.

In searching for a suitable waveform, one might consider the possibility of using a signal with a thumbtack-type ambiguity function, that is, a sharp central spike on a uniformly low pedestal. However, since nearly all of the fixed volume of the ambiguity function is in the pedestal, it is often impossible to decrease the pedestal level to where an acceptable interference level results from the superposition of the pedestals connected with all scatterers within the clutter space. This difficulty suggests the opposite approach of concentrating the ambiguity function within a very narrow strip oriented such that it cuts through as little of the clutter space as possible. This is the technique to be described in more detail in the following.

The utility of concentrating the ambiguity function within a narrow strip is perhaps more apparent when the target is outside the clutter space (although it may, physically, be inside the clutter region). For example, targets at position B and C in Fig. 1 could be detected free from interference if the ambiguity function were confined as indicated in the figure. Of course, a target at position B is really outside the range interval of the clutter, and the use of a short pulse is obvious. For the dual case, a target at C, target and clutter do not share the same Doppler interval, so that clutter suppression is feasible with a band-limited signal. On the other hand, if the target is at D, filtering in either time or frequency separately is not adequate. Combined filtering in delay and Doppler is necessary, in which case the strip cannot be parallel to one of the axes. The problem thus is to design ambiguity functions that are confined to a strip having arbitrary orientation in the delay-Doppler plane.

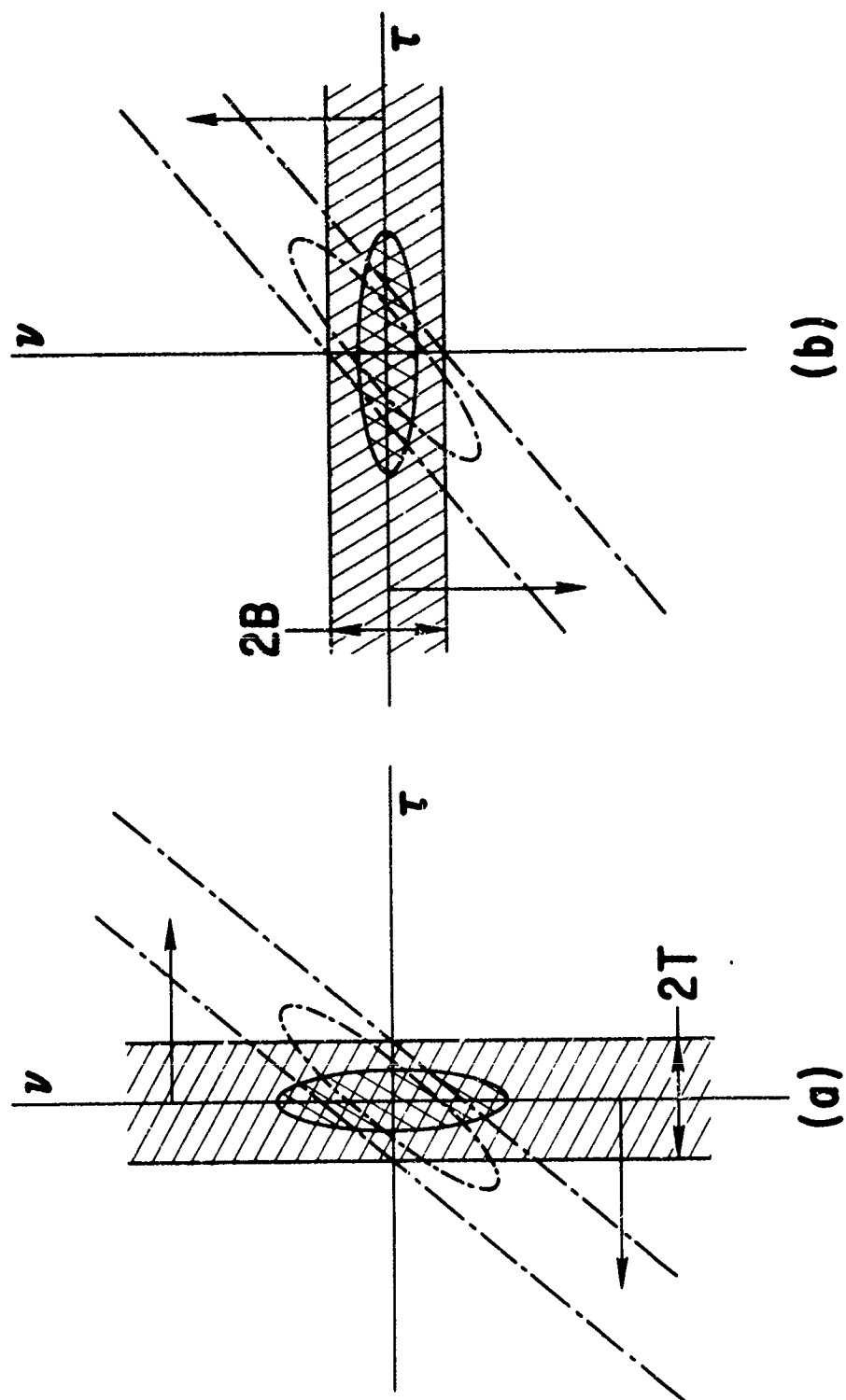


Fig. 2. Generation of maximally confined ambiguity functions.

III. SIGNALS WITH STRICTLY CONFINED AMBIGUITY FUNCTIONS

The objective is to find waveforms whose ambiguity functions are confined to a strip whose orientation can be chosen at will. With $\mu(t)$ the complex modulation function and $M(f)$ its Fourier transform, the ambiguity function may be defined as

$$\chi(\tau, \nu) = \int_{-\infty}^{\infty} \mu(t) \mu^*(t - \tau) e^{j2\pi\nu t} dt, \quad (1)$$

$$\chi(\tau, \nu) = \int_{-\infty}^{\infty} M^*(f) M(f - \nu) e^{j2\pi\tau f} df, \quad (2)$$

with τ the differential delay and ν the differential Doppler shift. We note that the desired ambiguity function can be obtained from that of a time-limited signal by shearing parallel to the delay axis (Fig. 2a). Formally, we are replacing in $\chi(\tau, \nu)$ the variable τ by $\tau + p\nu$, so that the former Doppler axis ($\tau = 0$) becomes the axis $\tau + p\nu = 0$. Substitution into Eq. (2) shows that the corresponding change in the signal is from $M(f)$ to $M(f)e^{-j\pi p f^2}$. This is one of Siebert's theorems concerning the ambiguity function [8]. The desired signal thus is generated by the introduction into a time-limited pulse of quadratic phase modulation in the frequency domain. In other words, the pulse is simply passed through a dispersive network. Alternatively, we can shear the ambiguity function of the band-limited pulse parallel to the Doppler axis, which means replacing in $\chi(\tau, \nu)$, the

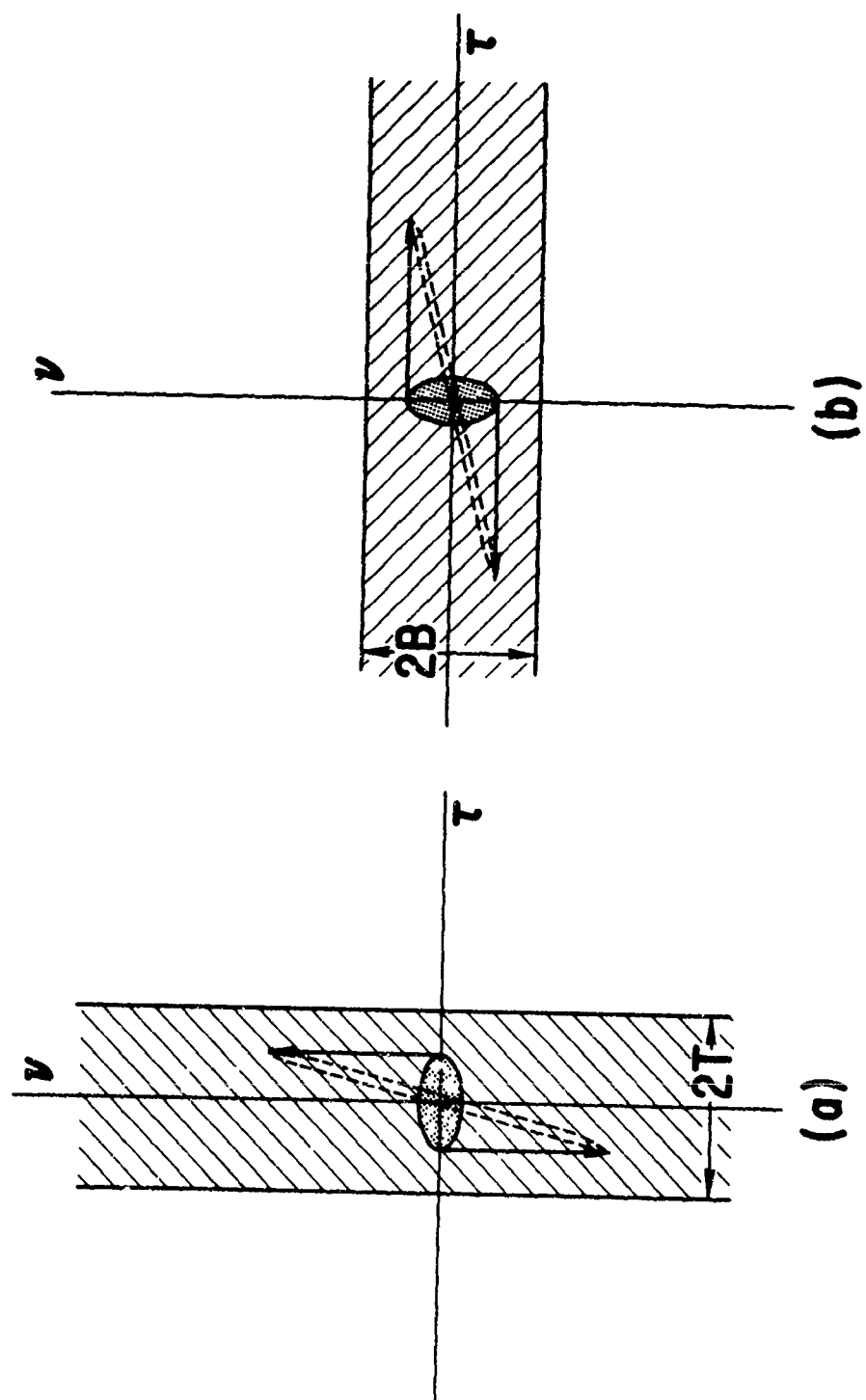


Fig. 3. Ambiguity function of the Chirp signal.

variable ν by $\nu + k\tau$. The corresponding change in the signal is from $\mu(t)$ to $\mu(t)e^{j\pi kt^2}$, which means introduction of the quadratic phase in the time domain, that is, active frequency modulation.

The reader familiar with pulse compression technology will be tempted to conclude that the above signal is the common Chirp signal, or linear FM pulse. Actually, there is an important difference between our signal and the Chirp waveform. With the operations indicated in Fig. 2, the quadratic phase is introduced in the domain in which the signal is unlimited. This contrasts with the Chirp signal, which is generated by modulating in the domain in which the signal is limited. The result is a shearing of the ambiguity function as shown in Fig. 3, where the two cases represent the active and passive generation methods. Only the main response peak is slanted, while the strip does not change its orientation and is much broader than the main peak.

From the fact that the modulation must be performed in the very domain in which the signal is unlimited, we immediately can conclude that the desired objective of confining an ambiguity function to a slanted strip is not attainable in practice. We can, however, approximate the waveform, and this will be considered later. For the time being, we ignore this problem and consider the different types of ideally confined ambiguity functions.

The method illustrated in Fig. 2 evidently yields a strip width of the same order as the width of the main response peak. Thus, for fixed width of this peak (and fixed close-target separability), the ambiguity function is

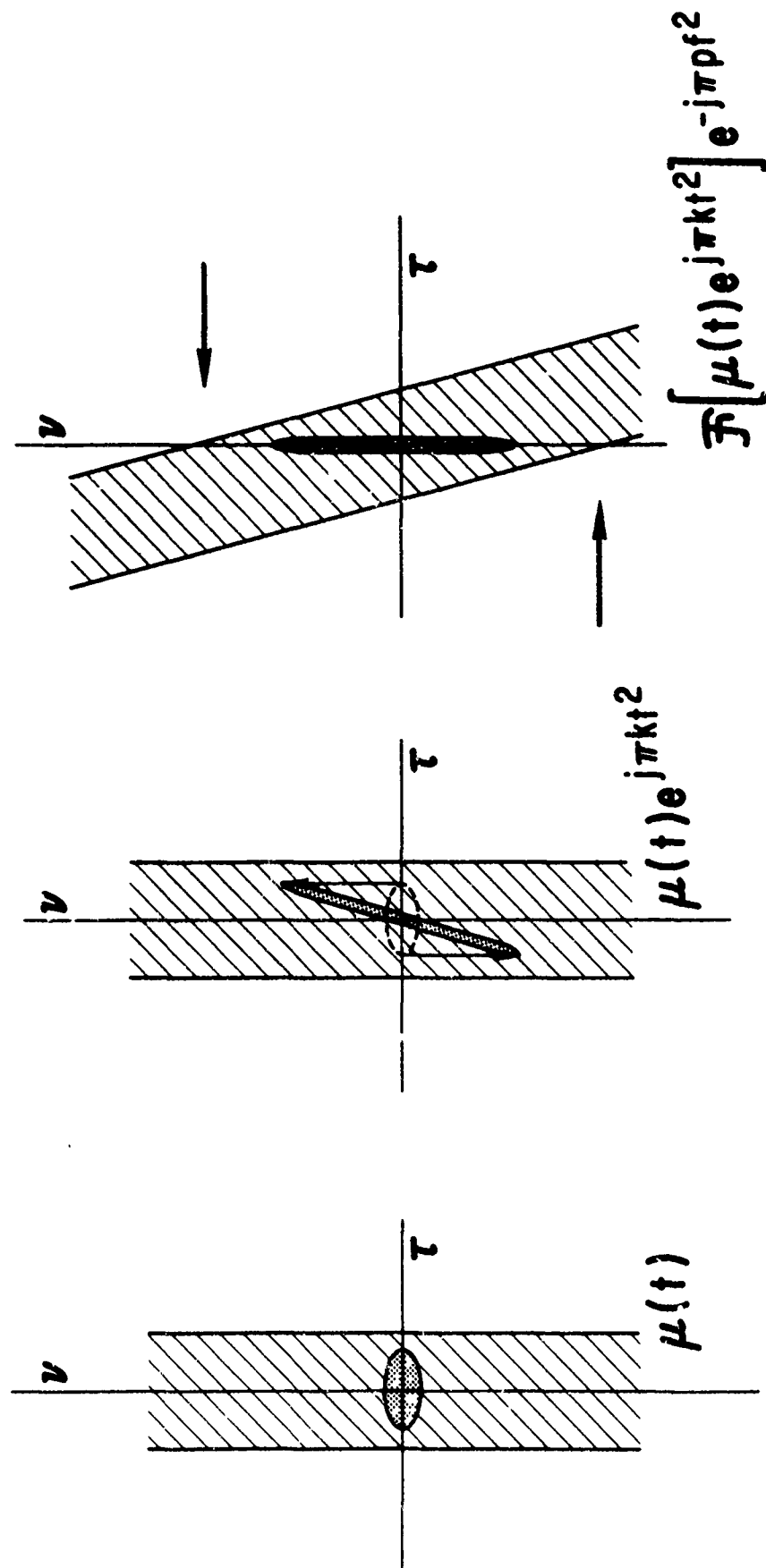


Fig. 4. Confinement of the ambiguity function to a slanted strip without linear FM.

confined to the narrowest possible strip. The waveform is a pulse compression signal with quadratic phase modulation. The question arises as to whether the strip can be slanted off the axis without the use of pulse compression. The corresponding, somewhat more complicated generation process is indicated in Fig. 4, starting from a time-limited pulse. In a first step, we introduce linear FM in the manner used with the active generation process of the Chirp waveform. This signal then is passed through a filter with proper quadratic phase which removes the linear FM. In the ambiguity diagram, this involves shearing parallel to the delay axis and produces the desired signal of unity time-bandwidth product and confined ambiguity function. The dual procedure for the band-limited pulse is obvious.

The above discussion establishes the fact that it is possible to rotate the strip independently of the main response peak. A comparison of Figs. 2 and 4 shows the following difference between a linear FM signal and a signal of unity time-bandwidth product. In the first case, the strip is about as wide as the main response peak in its narrow dimension. For the "simple" waveform of unity time-bandwidth product, on the other hand, the boundary lines for the strip are being rotated such that they remain essentially tangential to the main peak. The width of the strip thus changes with orientation. Only for the two special cases of time-limited or band-limited signals is the width of the strip given by the axes of the main response ellipse.

Rotation of the strip evidently is equally feasible if the starting signal is a pulse compression waveform. If, in Fig. 2, the main response width is narrowed, the figure serves as an illustration of the method of confining the ambiguity function to a strip wider than the main response peak. The associated waveform has a strong linear FM component superposed on the original modulation. In the same manner, Fig. 4 can be interpreted as a method of generating such an ambiguity function with a waveform without linear FM component.

IV. EFFECT OF SIGNAL TRUNCATION

We now investigate the effect on the ambiguity function of truncating the signal in the domain in which the modulation is applied. Consider the initial pulse to be time-limited with modulation function $\mu(t)$ and spectrum $M(f)$. Truncation of the spectrum to a band of width B gives a new spectrum $M_B(f) = \text{rect}(f/B) M(f)$, where $\text{rect}(x)$ is unity for $|x| \leq 0.5$ and zero otherwise. From Eq. (2), the new ambiguity function is

$$\begin{aligned} \chi_B(\tau, \nu) &= \int_{-\infty}^{\infty} \text{rect}\left(\frac{f}{B}\right) M^*(f) \text{rect}\left(\frac{f - \nu}{B}\right) M(f - \nu) e^{j2\pi f\tau} df \\ &= \left[\int_{-\infty}^{\infty} \text{rect}\left(\frac{f}{B}\right) \text{rect}\left(\frac{f - \nu}{B}\right) e^{j2\pi f\tau} df \right] \star \left[\int_{-\infty}^{\infty} M^*(f) M(f - \nu) e^{j2\pi f\tau} df \right] \\ &= \chi_{\text{rect}}(\tau, \nu) \star \chi(\tau, \nu) \quad , \end{aligned} \quad (3)$$

where the star denotes convolution in τ ; $\chi_{\text{rect}}(\tau, \nu)$ is the ambiguity function of a rectangular spectrum of width B ; and $\chi(\tau, \nu)$ is the ambiguity function of the original, time-limited signal. The original ambiguity function $\chi(\tau, \nu)$ is limited in the τ domain, but $\chi_{\text{rect}}(\tau, \nu)$ is not. Hence, the convolution operation of Eq. (3) spreads the ambiguity function over the entire τ domain. The smaller the bandwidth B , the wider $\chi_{\text{rect}}(\tau, \nu)$, and hence $\chi_B(\tau, \nu)$. A similar result obviously applies to the dual case of a band-limited signal that is truncated in the time domain when modulated.

The question of interest is to what degree the ambiguity function can be confined if the signal is truncated in the critical domain, or, equivalently, how much the ambiguity function is spread out in the domain in which it was confined if the signal is truncated. For clutter suppression, a suitable measure of the performance degradation is the total volume outside the original strip. This volume is found by integrating over $|\chi_B(\tau, \nu)|^2$ to the right side of the right boundary line and doubling the result. Since the strip is slanted and the ambiguity function is that of a linear FM signal, the calculation may seem rather complicated. However, because of the fact that $\chi_B(\tau, \nu)$ is obtained by shearing of the ambiguity function of an unmodulated pulse, it is evident that one can find the volume by performing the calculation on the unmodulated signal. If the original signal is time-limited, as assumed here, the shearing occurs parallel to the delay axis, so that the volume outside a strip of width τ_0 in delay is

$$V = \int_{\tau_0/2}^{\infty} d\tau \int_{-\infty}^{\infty} |\chi(\tau, \nu)|^2 d\nu \quad . \quad (4)$$

This relation applies for any ambiguity function associated with a constant-carrier signal; in particular, it applies for the ambiguity function after band-limiting, $\chi_B(\tau, \nu)$. If the dual approach were used, choosing a band-limited signal as the original waveform and then truncating in the time domain, one could calculate the volume more conveniently on the basis of the strip width ν_0 in Doppler.

Actually, the volume outside a particular strip $|\tau| \geq \tau_0/2$ is not of special interest. Suppose, for example, that the effect of truncation were a simple widening of the strip. The resulting ambiguity function would be just as useful for clutter suppression, since the widening of the strip could be offset by a shortening of the original signal duration. What is really of interest, then, is the degree of volume concentration, as measured by the volume outside an arbitrary strip as a function of the strip width. Thus we must consider τ_0 in Eq. (4) as a variable and calculate $V(\tau_0)$.

If the definition of the ambiguity function is used in Eq. (4), the inner integral can be expressed in terms of the squared modulus of the signal as

$$\int_{-\infty}^{\infty} |\chi(\tau, \nu)|^2 d\nu = \int_{-\infty}^{\infty} |\mu(t)|^2 |\mu(t - \tau)|^2 dt \quad (5)$$

Band-limiting of the signal means that $\mu(t)$ in Eq. (5) is replaced by its band-limited version

$$\begin{aligned} \mu_B(t) &= \int_{-\infty}^{\infty} \text{rect}\left(\frac{f}{B}\right) M(f) e^{j2\pi ft} df \\ &= \int_{-\infty}^{\infty} \mu(\xi) \frac{\sin \pi B(t - \xi)}{\pi(t - \xi)} d\xi \end{aligned} \quad (6)$$

The remaining integration in τ in Eq. (4) will usually have to be done numerically. If a band-limited signal is chosen as a start and truncation is performed in the time domain, the dual relation for integration over τ is given by

$$\int_{-\infty}^{\infty} |\chi(\tau, \nu)|^2 d\tau = \int_{-\infty}^{\infty} |M(f)|^2 |M(f - \nu)|^2 df \quad ,$$

with the spectrum of the time-truncated waveform

$$\begin{aligned} M_T(f) &= \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{T}\right) \mu(t) e^{-j2\pi ft} dt \\ &= \int_{-\infty}^{\infty} M(\eta) \frac{\sin \pi T(f - \eta)}{\pi(f - \eta)} d\eta \quad . \end{aligned}$$

V. THE RECTANGULAR PULSE

The only specification made for the "ideal" signal was that it be time-limited, so that its ambiguity function is confined in τ ; otherwise, the shape of the pulse is arbitrary. The shape of the pulse is significant insofar as it determines the envelope of the transmitted signal. For example, if a rectangular pulse is chosen, its spectrum has $(\sin x)/x$ form, which is then truncated. With the introduction of linear FM, the time function assumes approximately the truncated $(\sin x)/x$ form. The question of optimum pulse shape will not be investigated here; rather, we shall calculate the volume distribution for the waveform generated by phase modulating and truncating the rectangular pulse. The rectangular pulse is of interest because it represents the most severe form of time-limiting a signal.

The rectangular pulse has a modulation function $\mu(t) = \text{rect}(t/T)$ and a spectrum $M(f) = (\sin \pi T f)/\pi f$. From Eq. (6), the band-limited version is

$$\begin{aligned} \mu_B(t) &= \int_{-T/2}^{T/2} \frac{\sin \pi B(t - \xi)}{\pi(t - \xi)} d\xi \\ &= \frac{1}{\pi} \left\{ \text{Si} \left[\pi B T \left(\frac{1}{2} + \frac{t}{T} \right) \right] + \text{Si} \left[\pi B T \left(\frac{1}{2} - \frac{t}{T} \right) \right] \right\}, \end{aligned} \quad (7)$$

where $\text{Si}(x)$ is the sine integral defined by $\text{Si}(x) = \int_0^x [(\sin t)/t] dt$. The volume distribution of the ambiguity function may now be computed from Eqs. (4), (5), and (7). Only two numerical integrations are required, since the sine integral can be computed accurately and rapidly by a polynomial expansion [9].

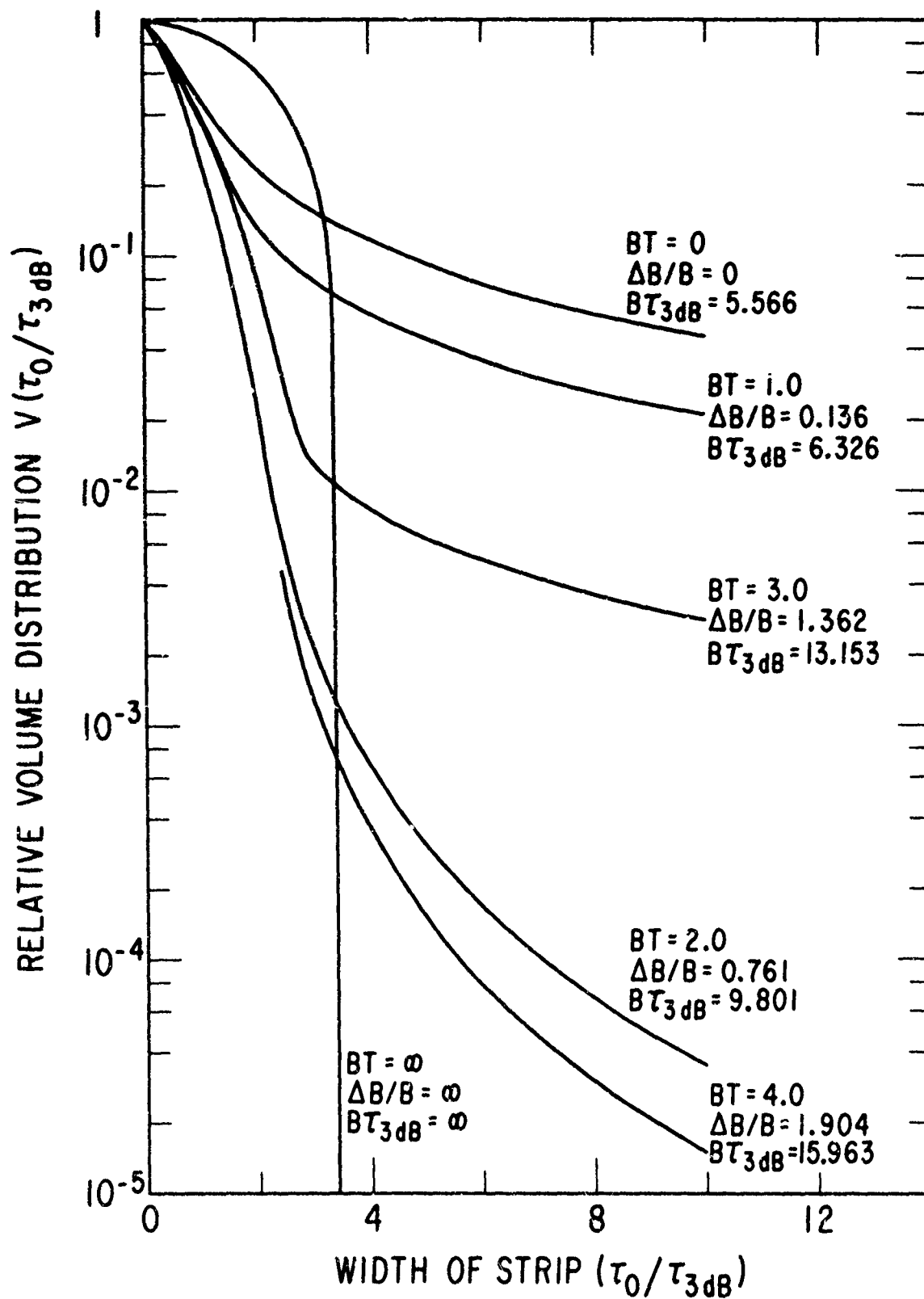


Fig. 5. Volume distribution for the signal with truncated $(\sin x)/x$ envelope.

Because the total volume under the ambiguity function is fixed, $V(\tau_0)$ can also be found from the volume within the strip, which is an easier calculation.

The parameters chosen for presentation of the results are the duration T of the original pulse and the width B of the truncated spectrum. Both the volume distribution and the shape of the spectrum are determined uniquely by the product BT . Since the truncated spectrum is given as $M_B(f) = \text{rect}(f/B) [(\sin \pi T f)/\pi f]$, a product $BT = 2$ means that the spectrum includes just the main lobe of the $(\sin x)/x$ function; for $BT = 4$ it includes the first sidelobe on both sides; for $BT = 6$, the first two sidelobes; and so forth. The rectangular spectrum of the Chirp signal results in the limit as $BT \rightarrow 0$.

Since changing B also changes range resolution, a comparison of volume distributions for various degrees of spectrum truncation is meaningful only on the basis of fixed close-target resolution. For this reason, the volume distribution is presented in Fig. 5 as a function of the normalized strip width τ_0/τ_3 dB, with the half-power width of the matched-filter response in τ chosen as the normalization constant. Curves are shown for several values of BT . Even integral values of BT represent spectra where the cutoff is at a zero crossing of the $(\sin x)/x$ function, while for odd integral values of BT , the cutoff is halfway between zero crossings. Truncation at the zeros is seen to give appreciably better performance.

A more detailed investigation shows that the zeros represent local optima for the truncation of the spectrum. This is not surprising. Taylor [10] found that the far-out sidelobes of a linear antenna are determined principally by the behavior of the illumination function at the edges of the

antenna. The far-out sidelobes are lower for an illumination function that goes to zero linearly at the edge than for one with a discontinuity at the edge. In general, the sidelobes are lower yet for an even more gradual approach to zero at the antenna edge. From Eq. (5), we must expect the same behavior for the volume of the ambiguity function. Thus, the volume should be more highly concentrated for truncated at zeros of the spectrum, and the farther out the zero the better.

Figure 5 shows that the volume outside a given strip decreases substantially as the bandwidth is increased to include more sidelobes of the spectrum. The penalty is the increase in signal bandwidth. In contrast to the Chirp signal, for which the band is uniformly filled with energy, we are now widening the band without putting substantially more energy into it. In other words, the bandwidth is increased without significant improvement of range resolution performance. The added band is used for suppression of the clutter rather than for improved close-target separability. We indicate this trade-off between target separability and clutter suppression in Fig. 5 by giving the values for the relative bandwidth increase compared to the Chirp signal necessary to achieve a given clutter reduction. Basis for comparison is a fixed half-power width of the ambiguity function on the τ axis.

VI. THE GAUSSIAN PULSE

In truncating the spectrum of the ideal waveform, we are removing the time-limitation of the signal and, hence, the strict confinement of the ambiguity function. The fact that we have started from the ideally time-limited signal thus has no real significance. This raises the question as to what performance is achievable if the original signal, before spectrum truncation, is unlimited in time. We shall investigate the gaussian pulse, because it is both representative for unlimited signals and readily tractable. Again, the phase function of the pulse will be quadratic if the strip to which the ambiguity function is essentially confined is to be slanted. The difference between the rectangular and the gaussian pulse thus is merely in their envelopes, and these signals should be fairly representative of that which is achievable with other envelope shapes.

In calculating the volume distribution, we can ignore again the phase modulation, as in the case of the rectangular pulse. The spectrum of the gaussian pulse can be taken as

$$M(f) = e^{-(f/\sigma)^2/2} \quad , \quad (8)$$

where the rms width 2σ is inversely proportional to the rms width of the signal. The band-limited signal is now given by

$$\mu_B(t) = 2 \int_0^{B/2} e^{-(f/\sigma)^2/2} \cos 2\pi f t \, df \quad , \quad (9)$$

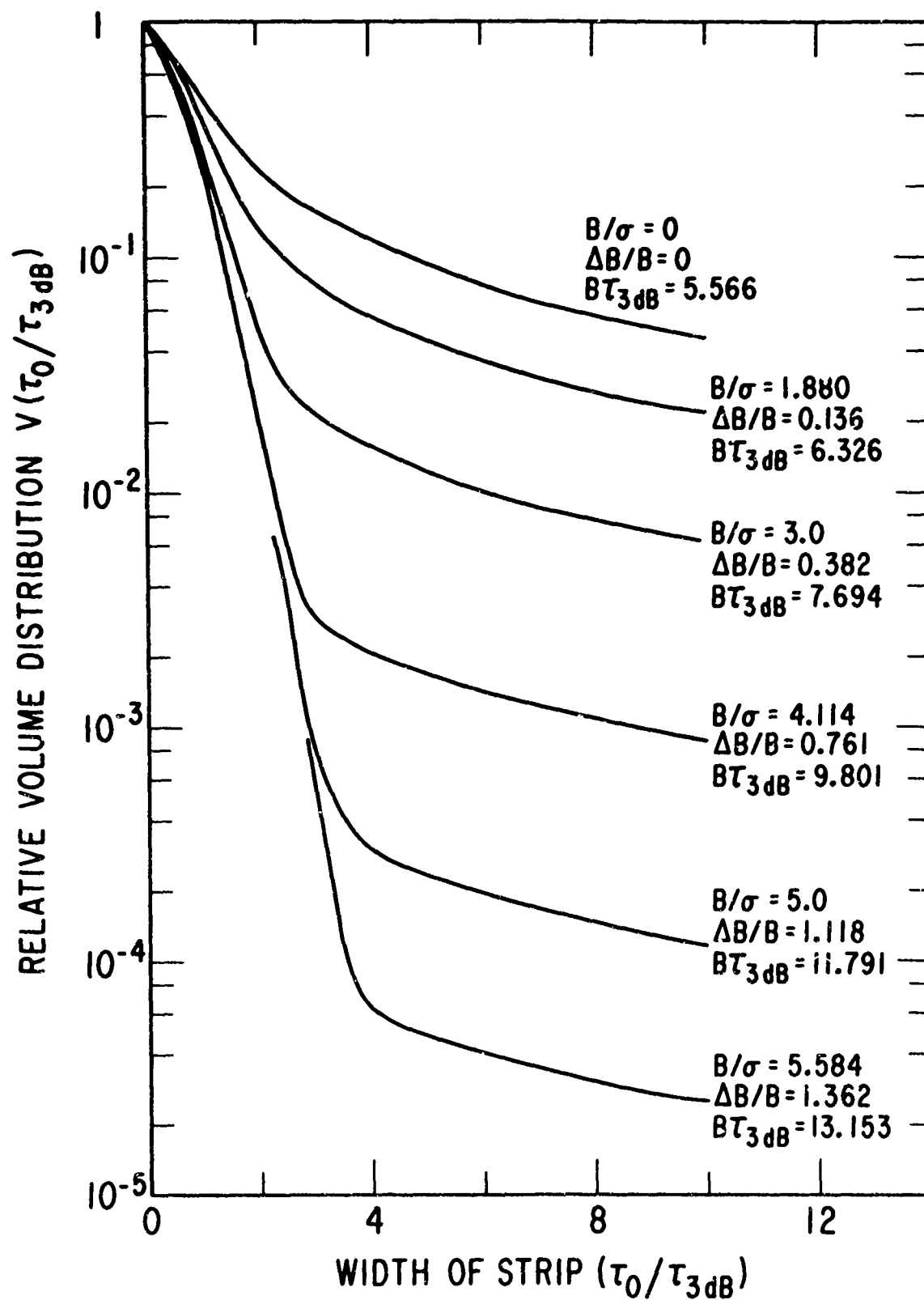


Fig. 6. Volume distribution for the signal with truncated gaussian envelope.

and the corresponding ambiguity function [Eq. (2)] by

$$\chi(\tau, \nu) = 2 e^{-(\nu/\sigma)^2/4} \int_0^{(B-\nu)/2} e^{-(f/\sigma)^2} \cos 2\pi f\tau \, df \quad (10)$$

Both Eqs. (9) and (10) can be expressed in terms of the error function of complex arguments, and a very accurate and rapid computer solution is possible [11, 12]. The volume distribution is found either by means of Eqs. (4), (5), and (9), or by direct volume integration of Eq. (10). Both methods involve two numerical integrations.

The results are shown in Fig. 6 for various values of B/σ . The width of the strip τ_0 has been normalized again to the half-power width of the ambiguity function on the τ axis. Also shown is the relative bandwidth increase over that for a rectangular spectrum necessary to reduce the volume outside the strip for fixed close-target separability in τ . For better comparison, some of the values of B/σ were chosen so that the resulting values of $\Delta B/B$ correspond to the values of $\Delta B/B$ in Fig. 5. In contrast to the results for the rectangular pulse, the improvement in volume concentration now is a monotonically increasing function of the bandwidth. This is to be expected since the gaussian spectrum is a smooth function, whereas the $(\sin x)/x$ spectrum of the rectangular pulse is not.

VII. RELATION OF VOLUME DISTRIBUTION TO SIGNAL ENERGY DISTRIBUTION

Upon examination of Eqs. (4) and (5), one would expect that if $\mu(t)$ had most of its energy concentrated within some interval, the ambiguity function would also have most of the volume concentrated within a strip limited in τ . Of course, these equations apply for the constant-carrier signal associated with the ambiguity function before shearing, that is, before the introduction of the quadratic phase in the frequency domain and the resulting change in the waveform. Nevertheless, the spectrum envelope of that waveform is the same as for the constant-carrier pulse, and because of the quadratic phase, the envelope in the time domain has approximately the same shape, at least when the time-bandwidth product is large. This means that the relation between volume of the ambiguity function and signal energy for the constant-carrier pulse could be used to determine pulse shapes that are well suited for concentrating the volume of the ambiguity function.

An approximate relationship between the ambiguity volume distribution and the signal energy distribution can be derived easily for the constant-carrier signal. In Eq. (5), assume that τ is larger than the width of the main lobe of $|\mu(t)|$. If only a small fraction of the signal energy is contained in the tails of $\mu(t)$, the principal contribution to the integral of Eq. (5) occurs over the main lobes of $|\mu(t)|^2$ and $|\mu(t - \tau)|^2$. Both contributions are equal (for symmetrical signals) and can be approximated if one of the two functions of the integrand in Eq. (5) is taken as a rectangle,

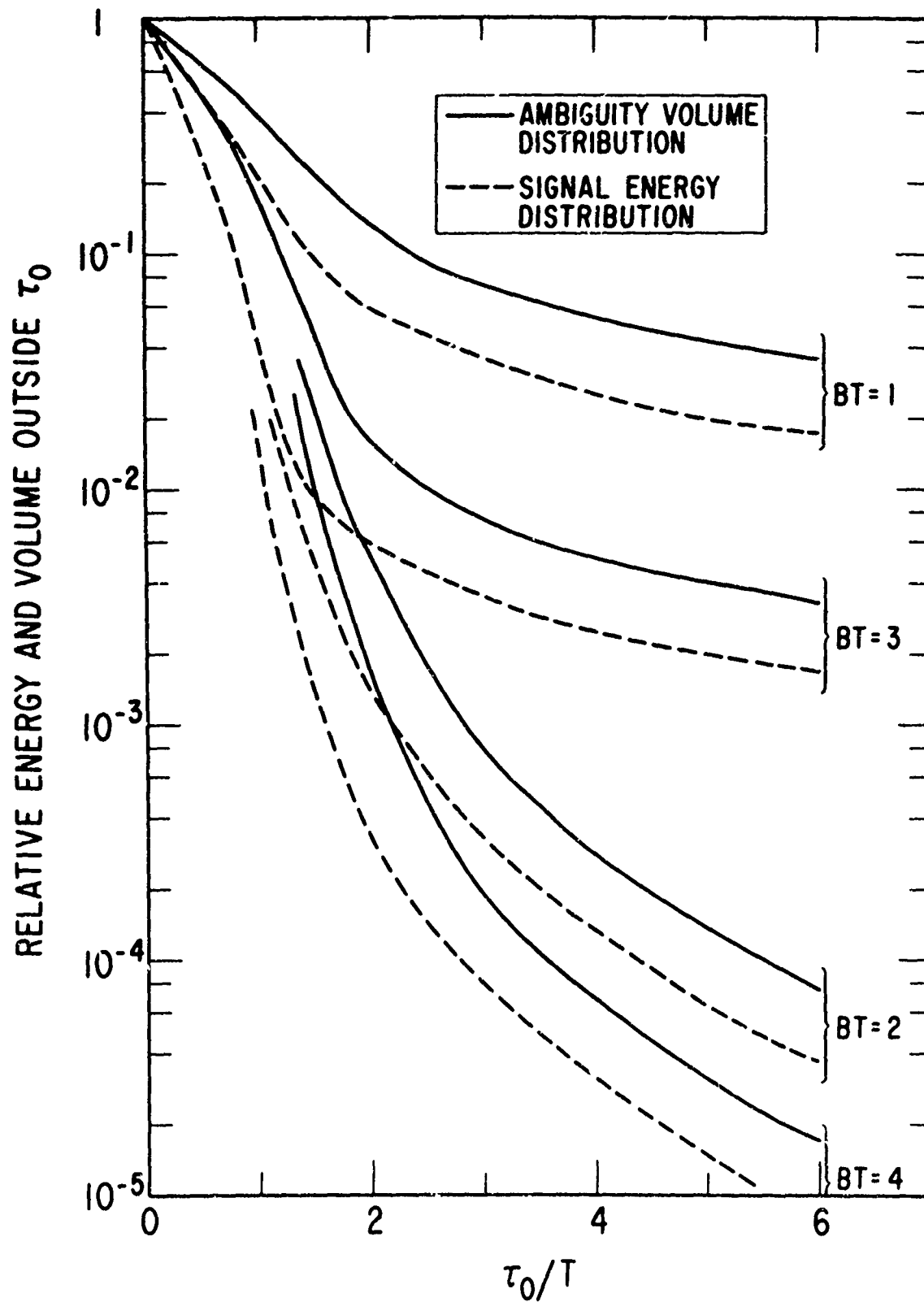


Fig. 7. Comparison of volume distribution and signal energy distribution.

$$|\mu(t)|^2 \approx \frac{1}{T'} \text{rect}\left(\frac{t}{T'}\right) \quad , \quad (11)$$

where the factor $1/T'$ was included for normalization of the signal energy.

Equation (5) then becomes

$$\int_{-\infty}^{\infty} |\chi(\tau, \nu)|^2 d\nu \approx \frac{2}{T'} \int_{-\infty}^{\infty} \text{rect}\left(\frac{t - \tau}{T'}\right) |\mu(t)|^2 dt \quad . \quad (12)$$

If this approximation is used in Eq. (4), the volume is

$$\begin{aligned} V(\tau_0) &\approx \frac{4}{T'} \int_{\tau_0/2}^{\infty} d\tau \int_{-\infty}^{\infty} \text{rect}\left(\frac{t - \tau}{T'}\right) |\mu(t)|^2 dt \\ &\approx \frac{4}{T'} \int_{\tau_0/2}^{\infty} d\tau \int_{\tau - T'/2}^{\tau + T'/2} |\mu(t)|^2 dt \quad . \end{aligned} \quad (13)$$

The integral in t can be approximated further as $T' |\mu(\tau)|^2$, which is merely an averaging operation since integration over τ follows. The volume thus is given as

$$V(\tau_0) \approx 4 \int_{\tau_0/2}^{\infty} |\mu(t)|^2 dt \quad . \quad (14)$$

As seen from this result, the volume outside the strip of width τ_0 is approximately double the signal energy outside the interval τ_0 . The validity of this relation between volume distribution and signal energy distribution is confirmed by Fig. 7, which presents curves for the rectangular pulse.

VIII. CONCLUSIONS

As shown by the curves of Figs. 5 and 6, a modest increase in the signal band leads to a substantial decrease of the volume outside a given strip. By more or less filling a given signal band, we can thus trade close-target separability against target detectability in the background clutter. In those situations where the clutter is the larger problem, the trade-off can be very favorable. For modest increases in the signal bandwidth, the performance of the truncated $(\sin x)/x$ envelope is better than that of the gaussian envelope, while the reverse is true for large bandwidth increases. Aside from these differences in the performance of signals with different envelopes, the important point is that linear FM permits concentration of the volume of the ambiguity function to a high degree. This fact always has been evident as far as the main ridge of the ambiguity function is concerned. As was shown above, the concentration of the volume within the central ridge also permits us to reduce the sidelobes surrounding the ridge.

The practically oriented reader may wonder how much of the theoretical potential for volume suppression can be realized in practice. One might argue that with the switching of the transmitter the sidelobes can be truly eliminated if a short pulse is used, while low sidelobes for a pulse compression waveform are the result of cancellation effects that depend on hard-to-maintain tolerances and stabilities. However, even the short pulse has sidelobes if the filter cuts off the outskirts of the spectrum. In

fact, the difference between the performance of such a pulse and that of the pulse compression waveform is given by the accuracy with which the phase modulation can be generated and removed. Although it is true that any deviations from ideal equipment performance will result in sidelobes, we are dealing here with the average sidelobe level, which can be very much lower than the peak sidelobe level.

Finally, it is interesting to consider the relation of the method discussed above to the well-known problem of reducing the range sidelobes in a Chirp system. The customary procedure of mismatching the receiver for sidelobe suppression is effective only for Doppler shifts not exceeding a few percent of the signal bandwidth. For sidelobe reduction over a larger Doppler band, we must perform the "weighting" of the signal at both the transmitting and the receiving side. This would suggest a gaussian signal envelope. However, in principle, such a signal never can have an ambiguity function that is strictly confined to a strip, since this is achievable only when the waveform is strictly limited in one domain. Since it then must have sidelobes in the other domain, and for linear FM the envelopes in time and frequency domain have about the same shape, we find that a waveform whose ambiguity function is strictly confined must always have sidelobes. This is perhaps the reason for the good performance of the $(\sin x)/x$ signal, in spite of the fact that the energy in the outskirts decreases rather slowly for such a signal.

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13 ABSTRACT <p>Target detection in the clutter from extended fields of scatterers, such as the ground or the sea, is a common problem in radar. Having optimized the system configuration, the designer can further improve radar performance by adapting the transmitted waveform to the particular target environment. This problem is investigated here for detection of a single target in the presence of an extended clutter space. The paper considers the possibility of confining the matched-filter response in delay and Doppler, or ambiguity function, to a narrow strip with arbitrary orientation in the delay-Doppler domain. It is shown that strict confinement of the response is achievable only with waveforms that are unlimited in both time and frequency domain. In practice, efficient use of the frequency band requires that the spectrum be truncated, so that strict confinement of the response is not achievable. One finds that bandwidth can be traded against visibility in clutter. More generally, for a fixed bandwidth, the trade-off is between close-target separability and detectability in the clutter, which are the two tasks that together constitute the resolution problem. The paper illustrates the effects of spectrum truncation for the important case of maximum confinement of the ambiguity function.</p>		

KEY WORDS

Radar Signal Design
Clutter Suppression
Radar Ambiguity Functions
Ambiguity Function Volume Distribution

Abstract (Continued)